# Degree Distribution of Wind Speed Induced Recurrence Networks

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## Abstract

Time series analysis employing complex network has attracted the attention of researchers. Several network creation methodologies for evaluating a time series basic dynamics were recorded. In this work, the degree heterogeneity of the recurrence networks of wind speed variations was evaluated. The recent interest in studying wind speed dynamics is because it is a clean energy source. The growing global demand for sustainable energy production has led to studying various dimensions of wind speed analysis. For this research work, data was collected from the National Renewable Energy Laboratory for wind speed time series of 10-minute interval over one year in 2004. Degree of heterogeneity of recurrence networks of wind speed variations for quantifying the diversity for characterizing the type and range of interactions was evaluated. The number of edges that are incident to a node in a network is the node's degree. The degree distribution shows power-law conformance which is a specific property of a scale-free network.

***Keywords***: *Recurrence network, Wind speed time series, Degree distribution, Degree heterogeneity, Complex network*

## 1.0 Introduction

Time series analysis employing complex network is getting increased attention in the literature. Network creation methodologies for time series is an actualization of a dynamical system, and hence overview of a complex system can be acquired by assessing the time series it generates [1]. In addition to conventional methods of analysing time series, the last decade witnessed development of various methods of developing a complex network for studying the characteristics of a data set [2]. Networks is an effective way for depicting and analysing the structure of complex systems. The first stage in this technique is to convert time-series data into a complex network [3]. Researchers employed variety of approaches to transform data sets into the complex networks. They

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include visibility graph method, transition matrix method, recurrence method and mutual information method. One of the novel and widely used methods is the recurrence network which analyses time series dynamic properties. In recurrence network construction, the time delay embedding approach converts time series into a set of data points in the attractor in m-dimensional state space. Each vector point of the attractor is considered a network node. Two nodes are connected with an edge if the distance between them is less than or equal to the given threshold value [3]. The network thus constructed is called a recurrence network which is characterised by network parameters. The degree of a node is determined by the number of links that intersect it. The degree distribution of a recurrence network is a significant factor which distinguishes different topologies of complex networks [4]. It also provides information on the underlying dynamic system's nature.

Studying the characteristics of wind speed dynamics is essential for optimizing wind turbine production and transmission. Wind speed fluctuates at all-time scales due to meteorological reasons. Wind power is proportional to wind speed and hence is constantly influenced by wind speed fluctuations [5]. This paper presents analysis of degree heterogeneity of the recurrence network of wind speed data from 25 different locations to gain insight into the dynamic characteristics of wind speed variations.

## 2.0 Methodology

The attractor reconstruction for a measured time series x (t) network is given by,

$$\mathbf{x}(t) = (x(t), x(t+\tau), \ldots, x(t+(m-1)\tau)), \qquad (1)$$

where, m and $\tau$ are the dimensions of embedding and the time delay, respectively [3,6].

Considering the time series $x_1, x_2, x_3, x_4, x_5, x_6, x_7, \ldots,$

m-dimensional state vectors $\mathbf{x}(t)$ with time delay $\tau$ is constructed as:

$$\mathbf{x}(t) = (x(t), x(t+\tau), \ldots, x(t+(m-1)\tau)), \; t = 1, 2, 3....$$

If m = 8 and $\tau$ =1;

$$\mathbf{x}_1 = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$$

$$\mathbf{x}_2 = (x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) \text{ and so on.}$$

With the reconstructed phase space, each vector $\mathbf{x}(t)$ is treated as a node of a network. There is an edge if the Euclidean distance between nodes $\mathbf{x}_i(t)$ and $\mathbf{x}_j(t)$ is less than or equal to the threshold value $\varepsilon=2.7$; otherwise, nodes $\mathbf{x}_i(t)$ and $\mathbf{x}_j(t)$ are unconnected.

Distribution of degrees is one of the most critical traits of a complex network. The edge count of a node determines the degree of the network. In network studies, distribution of the degrees across the network is known as the degree distribution. Equation (2) defines the degree distribution of a complex network. Degree distribution P(k) is the fraction of the network's vertices with degree k [7] as given by equation (2):

$$P(k) = \frac{numberofnodeshavingdegreekinthenetwork}{networkofsize} \quad (2)$$

There are many ways to define the heterogeneity index which measures the degree of diversity in a network [8]. Heterogeneity index h for an N-node recurrence network is given by equation (3):

$$h^2 = \frac{1}{N}\sum_{kmin}^{kmax}\left(1 - P(k)\right)^2, P(k) \neq 0 \quad (3)$$

where, k is the node degree and P(k) is probability distribution [8]. Equation (3) is normalized as:

$$H_m = \frac{h}{1-\frac{3}{N}+\frac{N+2}{N^3}} \quad (4)$$

Heterogeneity measure and index are nearly equal for sufficiently large N ( N > 1000 ). If $H_m = 1$, the network is entirely heterogeneous and for a completely homogeneous network $H_m = 0$.

## 3.0 Results and Discussion

Data for a period of one year of wind speed time series at 25 locations was considered and evaluated for heterogeneity of recurrence network. The data was obtained from the National Renewable Energy Laboratory (http://www.nrel.gov) for wind speed time series of 10-minute intervals over one year of 2004. Recurrence network was constructed with m=8 as the embedding dimension and delayed time $\tau=1$ and heterogeneity of recurrence network was evaluated. Recurrence network of a typical wind speed time series (Fig.1). Heterogeneity of recurrence network indicates variation of local structures within the network. There is a probability of more than one cluster. The colours in Fig. 1 show different clusters of the network.

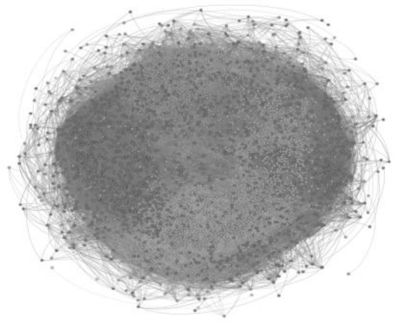

**Fig. 1.** Recurrence network of a typical wind speed

Degree distribution of wind speed recurrence network is shown in Fig. 2. Degree distribution of the recurrence network of a wind speed is reflected in the behaviour of power-law distribution. $P(k) = k^{-\gamma}$ is the power-law distribution for degree k and scaling factor $\gamma > 1$ [8]. If connectivity density function is directly proportional to power-law distribution, it results in a scale-free network [9].

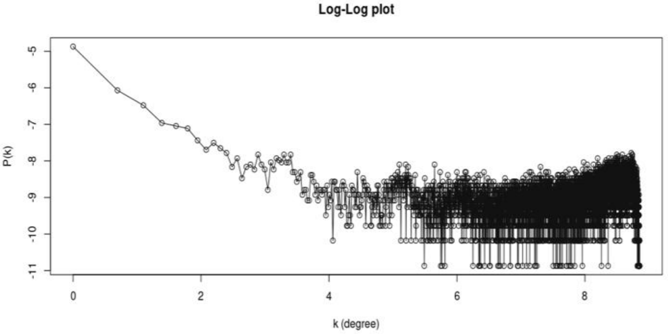


**Fig. 2.** Typical wind speed time series degree distribution of one year data

Jacob et al. [10] reported that the normalized heterogeneity index of the regular graph is zero and scale-free networks are approximately 0.3 for random graph considered. For a completely heterogeneous network, the index assumes a value one. Fig. 3 shows that the heterogeneity index of recurrence network of 25 sites. Fig. 3 shows that the heterogeneity values lie between 0.3 and 0.42, indicating that the heterogeneity of wind speed data of recurrence networks is similar to scale-free networks. It implies that the range of the computed values of heterogeneity index is similar to the reported value of a scale-free network.

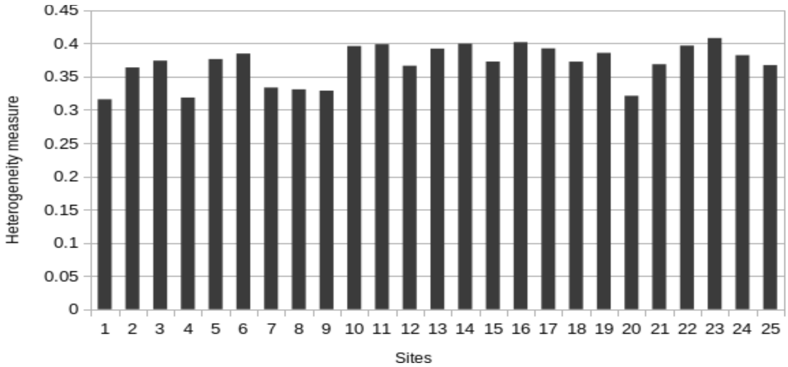


**Fig. 3.** Heterogeneity measures of recurrence networks of 25 sites

## 4.0 Conclusion

Degree of heterogeneity of recurrence network of wind speed data from a group of sites was studied to characterise the network. The degree distribution of recurrence network for the wind speed data showed power-law distribution. The non-zero values of heterogeneity index obtained showed that the recurrence network of time series was not homogeneous. The given wind speed networks were found to be scale-free. The relationship between the scale free nature of recurrence network and dynamical behaviour of wind speed variations can be further studied.

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